Quantum Creation of Black Hole by Tunneling in Scalar Field Collapse

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Abstract

Continuously self-similar solution of spherically symmetric gravitational collapse of a scalar field is studied to investigate quantum mechanical black hole formation by tunneling in the subcritical case, where, classically, the collapse does not produce a black hole.

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I. INTRODUCTION

Recently gravitational collapse of scalar fields have attracted much attention after the surprising discovery of critical phenomena by Choptuik [1]. The spherically symmetric collapse of a scalar field has been extensively investigated both analytically [2] and numerically [1,3]. The critical solution often displays self-similarity such that there exists a vector field, ξ, whose Lie derivative of the spacetime metric satisfies $L_\xi g = 2g$. The first known analytic solution of this kind was found by Roberts [4] in the context of counter-examples to cosmic censorship, and later discussed by Brady [5] in connection with critical behavior. Frolov [6] extended Roberts' work in general spacetime dimensions.

In the work of Roberts-Brady-Frolov [4–6] they considered subcritical, critical, and supercritical solutions depending on a parameter characterizing the solutions. In the supercritical solution a black hole is formed by the collapse, and in the critical case the spacetime is asymptotically flat but contains a null, scalar-curvature singularity. In the subcritical case, classically, the scalar field collapses, interacts and disperses leaving behind flat spacetime without forming a black hole.

In this paper we reconsider the subcritical case to investigate the quantum mechanical black hole formation by tunneling. Although the flux of the scalar field is not intense enough to classically form a black hole, it is possible to tunnel through the effective potential barrier and to continue collapsing to form a black hole. In the classically forbidden region we shall solve the field equations in Euclidean spacetime, whereas Roberts, Brady, and Frolov considered only spacetime with Lorentzian signature. We shall show that the black hole does form in the subcritical case through the tunneling mechanism. The mass of the black hole turns out to be infinity due to the self-similar nature of the solution. But one may view the solution as a near-critical solution with a finite mass in the spacetime region of a scale much below the relevant large correlation length.

In the section II, we present the field equations and effective one particle formalism, and in the section III we consider solutions in the Lorentzian and Euclidean regions of spacetime. There are two kinds of Lorentzian spacetime region : one before tunneling which is the one studied by Roberts, Brady, and Frolov. The other is the region after tunneling. In the Euclidean region an instanton type solution is obtained. In the section IV we join continuously the above three regions to treat black hole formation by quantum tunneling of the subcritical collapse of a scalar field. In the last section, we discuss similar problems in general dimensions.

II. EQUATIONS OF SPHERICALLY SYMMETRIC SELF-SIMILAR FIELDS

We study the minimally coupled scalar field in the four dimensional spacetime whose action is

$$S = \frac{1}{16\pi}\int_M d^4x\sqrt{-g}\ [R - 2(\bigtriangledown\phi)^2] + \frac{1}{8\pi}\int_{\partial M} d^3xK\sqrt{-h} \tag{1}$$

where K is the trace of second fundamental form of the boundary. Since we are only interested in spherically symmetric solutions, we can reduce the action as

$$S_{sph} = \frac{1}{4}\int d^2x\sqrt{-\gamma}\; r^2[R(\gamma) + \frac{2}{r^2}\{(\bigtriangledown r)^2 + 1\} - 2(\bigtriangledown\phi)^2], \tag{2}$$

where γ_{ab} is the metric in the remaining two-dimensional manifold. It is most easily handled in terms of null coordinates such that the metric is expressed as

$$ds^2 = -2e^{2\sigma}dudv + r^2 d\Omega^2, \tag{3}$$

where σ and r are functions of both u and v, and $d\Omega^2$ is the line element on the unit sphere. The Einstein-scalar field equations read

$$(r^2)_{,uv} = -e^{2\sigma} \tag{4.a}$$

$$2\sigma_{,uv} - \frac{2r_{,u}r_{,v}}{r^2} = \frac{e^{2\sigma}}{r^2} - 2\phi_{,u}\phi_{,v} \tag{4.b}$$

$$r_{,vv} - 2\sigma_{,v}r_{,v} = -r(\phi_{,v})^2 \tag{4.c}$$

$$r_{,uu} - 2\sigma_{,u}r_{,u} = -r(\phi_{,u})^2 \tag{4.d}$$

$$2\phi_{,uv} + (\ln r^2)_{,v}\phi_{,u} + (\ln r^2)_{,u}\phi_{,v} = 0, \tag{4.e}$$

where the last equation is the wave equation for the scalar field ϕ, and a comma (,) denotes a partial derivative.

In order to find a continuously self-similar solution, we impose the conditions as

$$\sigma(u,v) = \sigma(z), \quad r = -u\rho(z), \quad \phi = \phi(z), \tag{5}$$

where z is the scale-invariant variable defined as

$$z = -\frac{v}{u}\,. \tag{6}$$

The influx of the scalar field is turned on at the advanced time $v = 0$, so that the spacetime is flat to the past of this surface, and the initial conditions are specified by continuity. The region of interest is the sector $u < 0$, $v > 0$, where we choose signs such that $z > 0$, $\rho > 0$. With this choice of the self-similar metric and field, the equations (4.a)-(4.e) become

$$\rho^{''}\rho z + (\rho^{'})^2 z - \rho^{'}\rho = -\frac{1}{2}e^{2\sigma}, \tag{7.a}$$

$$\sigma^{''}z + \sigma^{'} + \frac{\rho^{''}z}{\rho} = -z(\phi^{'})^2, \tag{7.b}$$

$$\rho^{''} - 2\rho^{'}\sigma^{'} = -\rho(\phi^{'})^2, \tag{7.c}$$

$$\rho^{''}z + 2\sigma^{'}\rho - 2\sigma^{'}\rho^{'}z = -\rho z(\phi^{'})^2, \tag{7.d}$$

$$\phi^{''}\rho + 2\phi^{'}\rho^{'} = 0. \tag{7.e}$$

Prime denotes the derivative with respect to z.

FIGURES

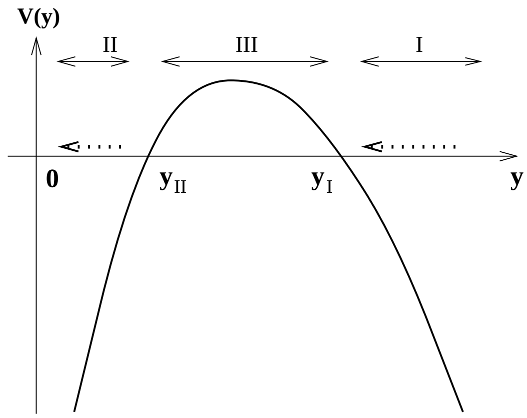

Figure 1: The height of the potential $V(y)$ is controlled by the value of c_0. The regions I, II and III are specified.

With new variables x and y, given by

$$x = \frac{1}{2}\ln z, \quad \rho = y(x)\sqrt{z}\,, \tag{8}$$

the above equations reduce to, with the metric factor $\sigma = 0$,

$$\dot{y}^2 = y^2 - 2 + \frac{c_0^2}{y^2}\,, \tag{9}$$

$$\dot{\phi} = \frac{c_0}{y^2}\,, \tag{10}$$

where c_0 is an integration constant, and dot denote derivative with respect to the new variable x.

The equation (9) for y formally describes motion of a particle with zero energy in a potential

$$V(y) = 2 - y^2 - \frac{c_0^2}{y^2}\,. \tag{11}$$

We are interested in the production of a black hole by quantum tunneling in the subcrictical case, $0 < c_0 < 1$, where the potential, only allows the classical motion of the particle starting from $y = \infty$ to a minimum value y_I. We call this the Lorentzian region I, where Roberts, Brady, and Frolov [4–6] provide explicit solutions in this region. There is another region, we call the Lorentzian region II, where classical motion is allowed, for $0 < y < y_{II}$. For quantum tunneling we need to consider the classically forbidden Euclidean region, $y_{II} < y < y_I$ as shown in the Fig. 1.

III. SOLUTION IN THE LORENTZIAN AND THE EUCLIDEAN SECTORS

(A) SOLUTION IN THE LORENTZIAN REGION I ($y_I \leq y < \infty$)

In this region where y is restricted as $y_I \leq y < \infty$, the solutions to the equations (9) and (10) are

$$y^2 = 1 + \sqrt{1 - c_0^2}\cosh 2x\,, \tag{12}$$

$$\phi = \frac{1}{2c_0}\ln\left(\frac{e^{2x}+\Gamma}{e^{2x}+\Gamma^{-1}}\right) - \frac{1}{c_0}\ln\Gamma\,, \tag{13}$$

where $0 < c_0 < 1$, and $\Gamma = \sqrt{(1-c_0)/(1+c_0)}$. We note that the minimum value of y is given by

$$y_I^2 = 1 + \sqrt{1-c_0^2}, \quad x = 0\,, \tag{14}$$

but there does not occur an apparent horizon as shown by Brady [5] and Frolov [6].

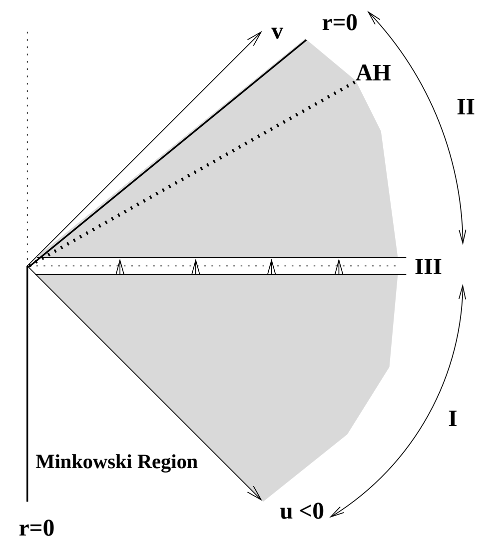

Figure 2: The spacetime diagram is obtained by joining the regions I and II. The presence of the region III is indicated by the strip with arrow, whose full structure does not appear in the diagram due to its Euclidean nature.

At the initial boundary ($v = 0, x = -\infty$) we choose the integration constant of ϕ such that $\phi(v = 0) = 0$, and the stress-energy tensor components are

$$T_{uu}|_{v=0} = 0, \quad T_{vv}|_{v=0} = \frac{1}{u^2(1-c_0^2)}, \quad T_{uv} = 0\,. \tag{15}$$

At the final boundary ($u = 0, x = +\infty$) the components of stress-energy tensor are

$$T_{uu}|_{u=0} = \frac{1}{v^2(1-c_0^2)}, \quad T_{vv}|_{u=0} = 0, \quad T_{uv} = 0\,, \tag{16}$$

which implies that the spacetime is flat for $u > 0$. However we will not consider the full region of $-\infty \leq x \leq \infty$, but limit to the region $-\infty \leq x \leq 0$, and connect this to the Euclidean tunneling solution with a smooth boundary condition as shown in Fig. 2.

(B) SOLUTION IN THE LORENTZIAN REGION II ($0 < y \leq y_{II}$)

In the region II where y is restricted as $0 < y \leq y_{II}$ the solutions to the equations (9) and (10) are

$$y^2 = 1 - \sqrt{1 - c_0^2} \cosh 2x \,, \tag{17}$$

$$\phi = \frac{1}{2c_0} \ln\left(\frac{\Gamma - e^{2x}}{\Gamma^{-1} - e^{2x}} \right) - \frac{1}{c_0} \ln \Gamma \,, \tag{18}$$

where $0 < \Gamma = \sqrt{(1 - c_0)/(1 + c_0)} < 1$ and $0 < c_0 < 1$. The maximum of y occurs at $x = 0$ such that

$$y_{II}^2 = 1 - \sqrt{1 - c_0^2}, \quad x = 0 \,. \tag{19}$$

There occurs black hole singularities when $y = 0$ ($r = y\sqrt{-uv} = 0$). Here, $\cosh 2x_0 = (z_0 + z_0^{-1})/2 = 1/\sqrt{1 - c_0^2}$, and so

$$z_0 = e^{2x_0} = \Gamma \quad or \quad \Gamma^{-1} \,. \tag{20}$$

The past singularity occurs at $z_0 = \Gamma$, while the future one at $z_0 = \Gamma^{-1}$.

Notice that the geometry does not extend to the light cones ($u = 0, \text{and } v = 0$) as the past light cone ($v = 0, z = 0$) is located at the z-value smaller than the allowed limit $z_0 = \Gamma$, and the future light cone ($u = 0, z = \infty$) has larger z-value than the allowed limit $z_0 = \Gamma^{-1}$. There occurs apparent horizons where $(\bigtriangledown r)^2 = 0$ which implies $\dot{y} + y = 0$. At the apparent horizon $y_{AH}^2 = c_0^2/2$, therefore

$$z_H = \sqrt{1 - c_0^2} \;\; \text{or} \;\; \frac{1}{1 - c_0^2} \,, \tag{21}$$

where the first one is the past apparent horizon of the white hole, while the second one is the future apparent horizon of the black hole to be formed.

(C) SOLUTION IN THE EUCLIDEAN REGION ($y_{II} \leq y \leq y_I$)

In this region the geometry is no longer Lorentzian, but becomes Euclidean, which is classically forbidden. We consider this region as a tunneling mediated solution, and for this purpose we have to change the signature of the geometry and rederive the field equations. Fortunately we only need to substitute the scale variable z as a complex number, namely,

$$z = e^{+2x} \longrightarrow z = e^{i2\theta} \,, \tag{22}$$

where θ is the polar angle in the $u - v$ plane. Denoting $\dot{y} = dy/d\theta$, $\dot{\phi} = d\phi/d\theta$ we obtain a new set of equations

$$\dot{y}^2 = -y^2 + 2 - \frac{c_0^2}{y^2} \,, \tag{23}$$

$$\dot{\phi} = i\frac{c_0}{y^2} \,, \tag{24}$$

where we note that the tunneling potential $V_{\rm tun}(y)$ is an upside down flip of the Lorentzian potential $V(y)$ in Eq. (11).

The solution to the Eqs. (23)-(24) are

$$y^2 = 1 + \sqrt{1-c_0^2}\cos 2\theta, \quad 0 < c_0 < 1\,, \tag{25}$$

$$\phi = \frac{i}{c_0}\tan^{-1}\left(\frac{c_0\tan\theta}{1+\sqrt{1-c_0^2}}\right) + \phi_0\,. \tag{26}$$

We note that y^2 is periodic in θ as it should be because it represents a metric in a plane. At $\theta = 0$, $y^2 = 1 + \sqrt{1-c_0^2}$ which is the same value as y_I^2 in the Lorentzian region I, so the Euclidean and the Lorentzian sectors are continuously connected. At $\theta = \pi/2$, $y^2 = 1-\sqrt{1-c_0^2}$ and this is equal to the y_{II}^2 in the inner region II, here again continuous patching of geometry is possible.

IV. BLACK HOLE FORMATION BY TUNNELING

In the subcritical case collapsing scalar fields approach only to y_I as x approaches to zero starting from $x = -\infty$ ($v = 0$), and bounce back to the future light cone ($u = 0$). However, this collapsing scalar field may tunnel through the Euclidean region and appear again with the value y_{II} at the matching point with the inner Lorentzian region. For this purpose we patch the three solutions to form a continuous complex spacetime as shown in the Fig. 2.

We recall that in the Lorentzian region I and II,

$$ds^2 = -2dudv + r^2d\Omega^2 = 2dw^2 - 2w^2dx^2 + r^2d\Omega^2\,, \tag{27}$$

where

$$u = -we^{-x},\ v = we^{x},\ w > 0\,, \tag{28}$$

$$r = wy = y\sqrt{-uv}\,. \tag{29}$$

The solutions to be patched are

$$y^2 = 1 + \sqrt{1-c_0^2}\cosh 2x, \ -\infty < x \leq 0, \ \ \text{(Region I)}\,, \tag{30}$$

$$y^2 = 1 - \sqrt{1-c_0^2}\ \cosh 2x,\ 0 \leq x < \infty, \ \ \text{(Region II)}\,. \tag{31}$$

Notice that at $x = 0$ these solutions are not continuously matched. What we interpret on this mismatch is that, as we can see in the Fig. 2, as x approaches zero the geometry come to the classical turning point and it tunnels through the potential barrier along the imaginary time while the real time remains at $x = 0$. In the imaginary time, which is resulted by the Wick rotation of the time x, the signature of spacetime becomes Euclidean. Here we use

$$ds^2 = 2dw^2 + 2w^2d\theta^2 + r^2d\Omega^2\,, \tag{32}$$

where $0 \leq \theta \leq 2\pi$, and x in Eq. (27) is substituted by $i\theta$. The solution in this region is

$$y^2 = 1 + \sqrt{1 - c_0^2}\,\cos 2\theta\,. \tag{33}$$

Note that y^2 in the region I and this region continuously joins at $x = 0$, $\theta = 0$, their value being $y_I^2 = 1 + \sqrt{1 - c_0^2}$. Now the geometry evolves through θ until θ approaches $\pi/2$, where y^2 approaches to the value $y_{II}^2 = 1 + \sqrt{1 - c_0^2}$, where it patches to the region II continuously at $x = 0$.

As shown in Fig. 2, in the spacetime region prior to $v = 0$ the geometry is flat Minkowski, and it evolves along x from $x = -\infty$ to $x = 0$ as the scalar fields collapse, then parts of the fields bounce back. But some may tunnel through the potential barrier evolving through the Euclidean geometry from $\theta = 0$ to $\theta = \pi/2$, and the fields reappear at the inner region where the spacetime geometry forms an apparent horizon at $x_{AH} = -\frac{1}{4}\ln(1 - c_0^2)$, and finally collapse to form a black hole at $x_{BH} = \frac{1}{4}\ln\Big((1 + c_0)/(1 - c_0)\Big)$ leaving a singularity ($r = 0$).

In order to evaluate the probability to form a black hole by quantum tunneling through the Euclidean barrier we follow usual methods of instanton action calculation. We recall that the action is

$$\begin{aligned} S_{sph} &= \frac{1}{4}\int d^2x\sqrt{-\gamma}r^2[R(\gamma) + \frac{2}{r^2}\{(\triangledown r)^2 + 1\} - 2(\triangledown\phi)^2] \\ &= \int \omega d\omega d\theta((\triangledown r)^2 + 1 - r^2(\triangledown\phi)^2) \end{aligned} \tag{34}$$

where we use the metric, and the coordinates

$$\gamma_{ab} = \begin{pmatrix} 2 & 0 \\ 0 & -2\omega^2 \end{pmatrix}, \quad u = -\omega e^x, \quad v = \omega e^{-x}\,. \tag{35}$$

Since $r = \omega y$, we can evaluate $(\triangledown r)^2$ and $(\triangledown\phi)^2$ as

$$(\triangledown r)^2 = \gamma^{ab}\partial_a r\partial_b r = \frac{1}{2}(y + \omega\frac{\partial}{\partial\omega}y)^2 - \frac{1}{2}\dot{y}^2\,, \tag{36}$$

$$(\triangledown\phi)^2 = \frac{1}{2}(\partial_\omega\phi)^2 - \frac{1}{2\omega^2}\dot{\phi}^2\,, \tag{37}$$

where $\dot{y} = \frac{\partial y}{\partial x}$, $\dot{\phi} = \frac{\partial\phi}{\partial x}$. The action reads explicitly

$$S_{sph} = \frac{1}{2}\int dq dx\left[-\dot{y}^2 + y^2\dot{\phi}^2 + 2 - y^2 + (2q\frac{\partial}{\partial q}y)^2 - y^2(2q\frac{\partial}{\partial q}\phi)^2\right]. \tag{38}$$

where $q \equiv \frac{\omega^2}{2}$ and we have dropped an irrelevant total derivative term.

Now we turn to the particle picture tunneling through the potential barrier as shown in the Fig. 1. Defining the momentum densities as

$$\pi_y = \frac{\partial\mathcal{L}}{\partial\dot{y}} = -\dot{y}, \quad \pi_\phi = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = y^2\dot{\phi}\,, \tag{39}$$

we obtain the effective superspace Hamiltonian density

$$\begin{aligned} \mathcal{H} &= \pi_y\dot{y} + \pi_\phi\dot{\phi} - \mathcal{L} \\ &= -\frac{\pi_y^2}{2} + \frac{\pi_\phi^2}{2y^2} - \frac{1}{2}\left[2 - y^2 + (2q\frac{\partial}{\partial q}y)^2 - y^2(2q\frac{\partial}{\partial q}\phi)^2\right]. \end{aligned} \tag{40}$$

In order to get the equivalent particle picture, one may consistently set $\frac{\partial \hat{\phi}}{\partial q}|\psi\rangle = \frac{\partial \hat{y}}{\partial q}|\psi\rangle = 0$ by demanding $|\psi\rangle$ to be a function only of the zero-mode variables, $\tilde{y} \equiv \int dq\, y(q)/\int dq$ and $\tilde{\phi} \equiv \int dq \phi(q)/\int dq$. Then diagonalizing π_ϕ by $|\psi\rangle = |\Phi\rangle e^{ic_0 \int dq \phi}$ with $\frac{\delta}{\delta \phi}|\Phi\rangle = 0$, the problem is further reduced to

$$
\begin{aligned}
& H|\Phi\rangle = 0 \\
& H = -\frac{1}{2K}\frac{\partial^2}{\partial \tilde{y}^2} + \frac{K}{2}\left[2 - \tilde{y}^2 - \frac{c_0^2}{\tilde{y}^2}\right] ,
\end{aligned}
\tag{41}
$$

where $K \equiv \int dq$. The effective action of the equivalent particle is then

$$
S_{eff} = \frac{K}{2}\int dx \left[\dot{\tilde{y}}^2 - 2 + \tilde{y}^2 + \frac{c_0^2}{\tilde{y}^2}\right]
\tag{42}
$$

with the Hamiltonian constraint $H \simeq 0$. Notice that the Euclidean version of this action gives the equation of motion of the particle given by Eq. (23). The tunneling probability can be evaluated within the standard WKB approximation scheme. Inserting the solution $y^2 = 1 + \sqrt{1 - c_0^2}\cos 2\theta$ with $x = i\theta$, and integrating from $\theta = 0$ to $\theta = \pi/2$ we get

$$
\begin{aligned}
S_{eff} &= K\int_0^{\frac{\pi}{2}} d\theta \left[\frac{(1-c_0)^2 \sin^2 2\theta}{1 + \sqrt{1 - c_0^2}\cos 2\theta}\right] \\
&= \int d\left(\frac{\omega^2}{2}\right)\frac{\pi}{2}(1 - c_0) ,
\end{aligned}
\tag{43}
$$

where we used $K = \int d\left(\frac{\omega^2}{2}\right)$ in the last line. It is not possible to use this effective action as it diverges unless we introduce a cut-off in the $\omega-$variable. As usual in the self-similar solutions this kind of divergences is unavoidable, so we introduce an arbitrary cut-off even though this is not allowed in the strict sense of self-similar solution. This divergence is an unnatural artifact of self-similarity as the black hole mass also diverges. With such a cut-off ω_c, we get the probability of tunneling, therefore of black hole production as

$$
P = e^{-2S_{eff}} = e^{-\frac{\omega_c^2}{2}\pi(1-c_0)} .
\tag{44}
$$

The probability of the bouncing without black hole formation is just $1 - P$. We note that in the limit of critical case ($c_0 \to 1$) we get the expected value $P = 1$.

Here the consideration of relative probability to the bouncing case does not remove the cut-off dependence contrary to the case of, for example, the free energy in the Schwarzschild-Anti-de-Sitter relative to pure Anti-de-Sitter space [7]. In order to avoid this divergent action naturally we may have to solve the equations with scalar flux of finite duration only, but this would require numerical studies. One may interpret the solution as a near-critical solution with a finite mass and a finite probability of tunneling in the spacetime region of a scale much below the relevant large correlation length. Then the cut-off scale will be naturally controlled by the correlation length.

Brady [5] defined a local mass function $m(u, v)$ by

$$1 - \frac{m(u,v)}{r} = 2g^{uv} r_{,u} r_{,v} \tag{45}$$

which agrees with both the ADM and Bondi masses in the appropriate limits. We apply this formula to get the black hole mass formed after tunneling ($x \geq 0$) as

$$m(u,v) = \frac{c_0^2}{4}\frac{\sqrt{-uv}}{y} = \frac{c_0^2}{4}\left(\frac{-uv}{1-\sqrt{1-c_0^2}\cosh 2x}\right)^{\frac{1}{2}}, \tag{46}$$

and, along the apparent horizon ($z_H = 1/\sqrt{1-c_0^2},\ y_{AH}^2 = c_0^2/2$), we get

$$m_{AH} = \frac{v}{2\sqrt{2}} c_0 (1-c_0^2)^{\frac{1}{4}}, \tag{47}$$

which agrees with the Brady's result [5]. In order to compare with the Brady's calculation, one should note that his parameters(α, β) are related to c_0 as $\alpha = \beta = \frac{1}{2}\sqrt{1-c_0^2}$ giving an exponent $\frac{1}{2}$.

The discrepancy of the critical exponent from the Choptuik's (~ 0.37) may be understood as follows. In the near-critical interpretation with a finite but large correlation length scale, the apparent horizon will continue to develop in the outside region of the correlation length scale, and eventually match with the event horizon if one considers the influx of scalar field with a finite duration. The mass viewed in this outside region, then, is expected to change appreciably and may conform with the Choptuik's.

V. DISCUSSION

Frolov [6] extended the work of Roberts [4] and Brady [5] to the problems in n-dimensions, where the action is

$$S = \frac{1}{16\pi}\int d^n x \sqrt{-g}[R - 2(\bigtriangledown\phi)^2], \tag{48}$$

which reduces to the spherically symmetric action as

$$S_{sph} \propto \int d^2 x \sqrt{-\gamma} r^{n-2}[R(\gamma) + (n-2)(n-3)r^{-2}((\bigtriangledown r)^2 + 1) - 2(\bigtriangledown\phi)^2]. \tag{49}$$

With the self-similarity ansatz we introduce

$$d\gamma^2 = -2e^{2\sigma(z)} du dv,\ r = -u\rho(z),\ \phi = \phi(z), \tag{50}$$

and we get the effective particle motion as

$$\dot{y}^2 = y^2 - 2 + c_1 y^{-2(n-3)}, \tag{51}$$
$$\dot{\phi} = c_0 y^{-(n-2)}, \tag{52}$$

where

$$c_1 = \frac{2c_0^2}{(n-2)(n-3)} > 0\,. \tag{53}$$

Here as in the previous cases we use the definitions, $x = \frac{1}{2}\ln z$, and $\rho = \sqrt{z}y(x)$. The solution can be formally presented as

$$x = \pm \int \frac{dy}{\sqrt{y^2 - 2 + c_1 y^{-2(n-3)}}} + c_2\,, \tag{54}$$

which can be performed explicitly in terms of elliptic functions only in the dimension $n = 5$, and $n = 6$.

In the Euclidean sector, the solution becomes

$$\theta = \int \frac{dy}{\sqrt{2 - y^2 - \frac{c_1}{y^{2(n-3)}}}} + c_2\,, \tag{55}$$

where $0 \le \theta \le \pi$. The solution $y = y(\theta)$ is an instanton like solution. We may join the above solutions continuously to form a tunneling geometry. Even though the solution need not be periodic, it is an interesting observation that the period satisfies the condition,

$$\frac{2\pi}{\sqrt{2(n-2)}} \le T_n < \pi \quad (n \ge 5)\,. \tag{56}$$

Notice that the period is π for the four dimensional spacetime, but there are no periodic solutions between $5 \le n \le 9$, and yet there exists periodic solutions for $n \ge 10$. We do not know whether this is somehow connected with supergravity models of dimension ten or eleven.

As a passing remark we mention that the effective potential (23), $V(y) = y^2 + c_0^2/y^2 - 2$, is a potential of a three dimensional simple harmonic oscillator with an angular momentum c_0. It also reminds us the Calogero model [8] whose Hamiltonian is $H = p^2 + \lambda^2/q^2$. Our potential may be viewed as simple harmonic oscillator modified by Calogero potential such that solubility might be preserved.

Finally we should remind readers that our quantum black hole formation process is different from those black hole pair creations via instanton where gravitational collapse is not the driving mechanism [9]. Our geometry is more kin to the Hartle-Hawking's no boundary creation of the Universe [10], or Vilenkin's tunneling proposal [11]: our solutions with reversed time describes nothing but the creation of the self-similar universe by tunneling, although its relevance to cosmology needs to be clarified further.

ACKNOWLEDGMENTS

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